4317 Exam 2

Thursday, October 25, 2018 9:47 PM

Last Name:
First Name:
BU or BG:

Which 4 problems do you want graded?

Problem 1 (5 points): Let E be a compact metric space. Let $C(E)$ be the metric space of real valued continuous functions defined on E with the distance defined in class, i.e. $D(f,g) = max_{x\in E}|f(x) - g(x)|$. Let $p \in E$. We define $F: C(E) \to R$ by $F(f) = f(p)$. Prove that F is uniformly continuous.

Problem 2 (5 points): Show that the closed ball in $C([0,1])$ of center 0 and radius 1 is not compact.

Problem 3 (5 points): Let $f_n: [0,1] \to R$ be a sequence of an increasing functions. Assume that f_n converges to f pointwise. Prove that f if is increasing. If in addition f is continuous, prove that the convergence is uniform.

Problem 4 (5 points): Prove that if f is a differentiable real-valued function on an open interval in R then f is increasing if and only if f' is nonnegative at each point of the interval.

Problem 5 (5 points): Assume $f: R \to R$ is twice differentiable at $x = 1$, that $f'(1) = 0$ and that $f''(1) = 1$. Prove that $x = 1$ is a strict local minimum of f, i.e. there exist $\delta > 0$ such that $f(1) < f(x)$ for all x such that $0 < |x - 1| < \delta$.